A Study of a Transverse crack in Orthotropic and Layered Materials using both Analytical and Finite Element Techniques

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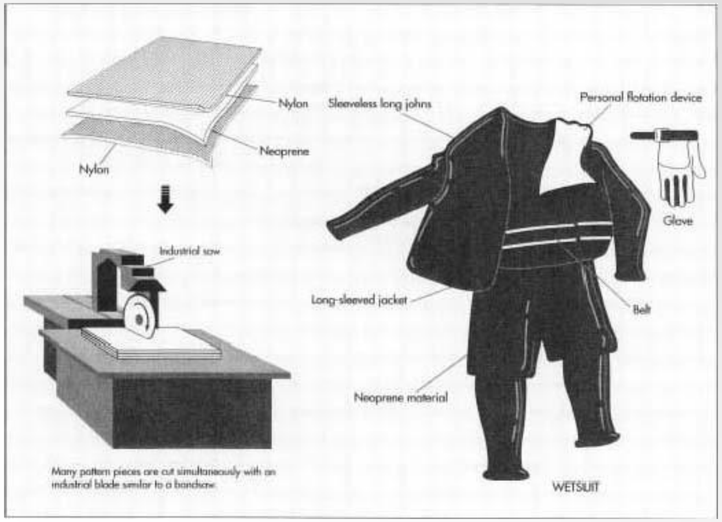
Agenda

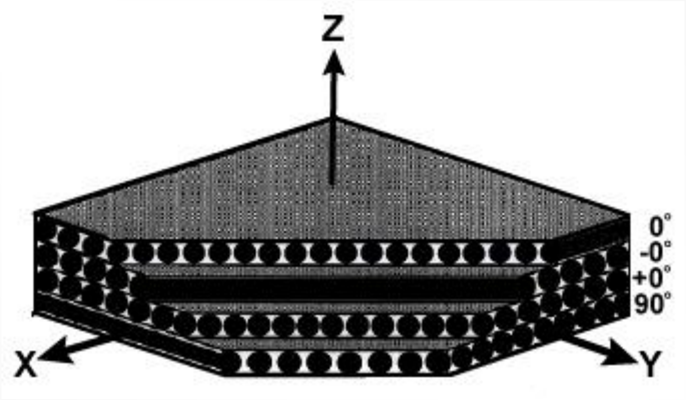
- Introduction
- Analytical Work
- Finite Element Work
- Conclusions
- Acknowledgements
- References

Introduction

Introduction - Motivation

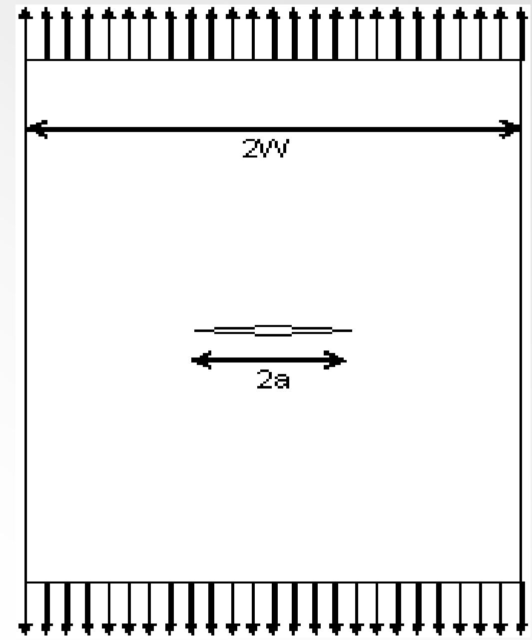
- Layered Materials are very common
- Most Fracture work has examined interlaminar cracks





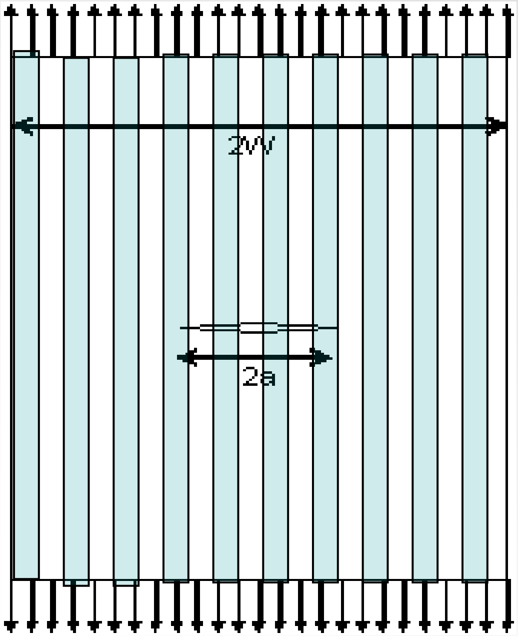
Introduction - Motivation

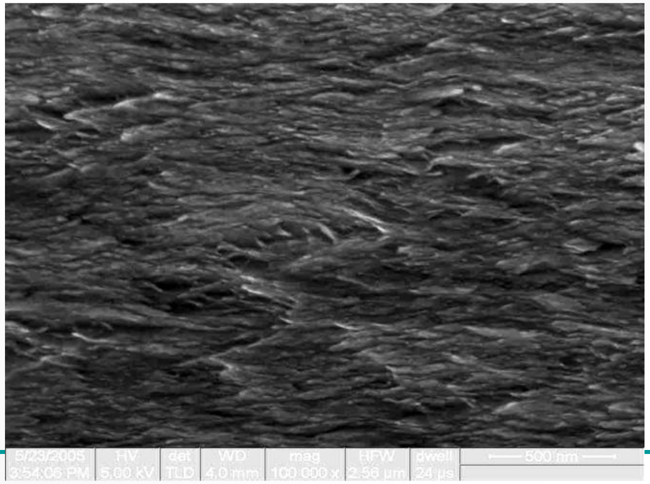
- Layered Materials are very common
- Most Fracture work has examined interlaminar cracks
- Very little done in cracks perpendicular to the layer (transverse)



Introduction - Motivation

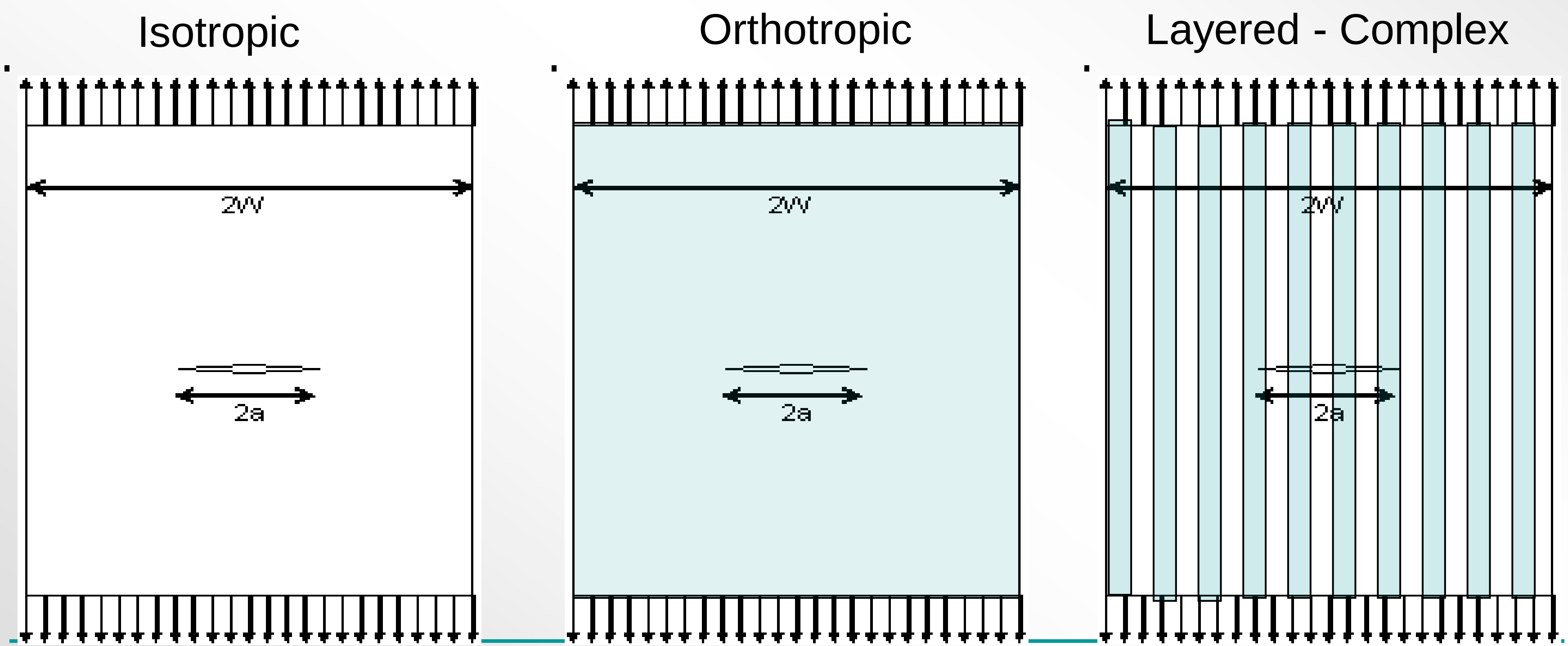
- Layered Materials are very common
- Most Fracture work has examined interlaminar cracks
- Very little done in cracks perpendicular to the layer (transverse)
- Of growing importance for nanostructured materials





Goals of This Research

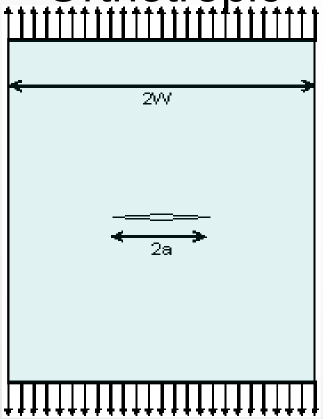
- Find the controlling parameters that affect crack propagation in a layered structure
- Use Analytical and Finite Element Techniques



Layered vs. Orthotropic

Hybrid Model (Sun)

Orthotropic



E1, E2, nu_21, G12

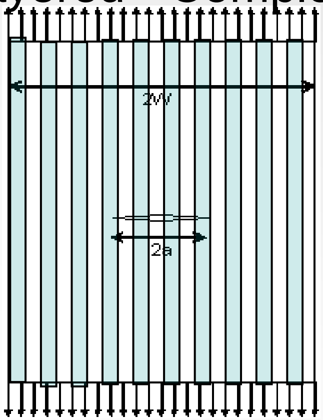
$$E_2 = c_a \cdot E_a + c_b \cdot E_b$$

$$\frac{1}{E_1} = \frac{c_a}{E_a} + \frac{c_b}{E_b} - \frac{c_a \cdot c_b \left(\nu_a \cdot E_b - \nu_b \cdot E_a\right)^2}{E_a \cdot E_b \cdot \left(c_a \cdot E_a + c_b \cdot E_b\right)}$$

$$\nu_{21} = c_a \cdot \nu_a + c_b \cdot \nu_b$$

$$\frac{1}{G_{12}} = \frac{c_a}{G_a} + \frac{c_b}{G_b}$$

Layered - Complex



Ea, Eb, nu_a, nu_b

Analytical Work

Stress Function for Anisotropic Materials

Equation	Isotropic [2]	Anisotropic [1]
Equilibrium	$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} = 0, \quad \frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} = 0$	
Stress Function	$\sigma_{xx} = \frac{\partial^2 \phi}{\partial y^2}, \quad \sigma_{yy} = \frac{\partial^2 \phi}{\partial x^2}, \quad \sigma_{xy} = -\frac{\partial^2 \phi}{\partial xy}$	
Compatibility	$\frac{\partial^2 \varepsilon_{xx}}{\partial y^2} + \frac{\partial^2 \varepsilon_{yy}}{\partial x^2} = -\frac{\partial^2 \gamma_{xy}}{\partial xy}$	
Governing Equation	$\nabla^4 \phi = 0$	$D_1 D_2 D_3 D_4 \phi = 0$

$$D_i = \frac{\partial}{\partial y} - s_i \frac{\partial}{\partial x}$$

Material Dependency

$$a_{11} s_i^{\,4} - 2a_{16} s_i^{\,3} + (2a_{12} + a_{66}) s_i^{\,2} - 2a_{26} s_i + a_{22} = 0$$

Stress Function for an Anisotropic Plate

Isotropic

$$\phi = \mathrm{Re}\left\{ \bar{z}\psi(z) + \chi(z) \right\}$$

Orthotropic

$$\phi = 2\,\mathrm{Re}\left\{ \psi(z_1) + \chi(z_2) \right\}$$

Stress Function for an Anisotropic Plate

Isotropic

$$\phi = \mathrm{Re}\left\{\bar{z}\psi(z) + \chi(z)\right\}$$

$z = x + iy$

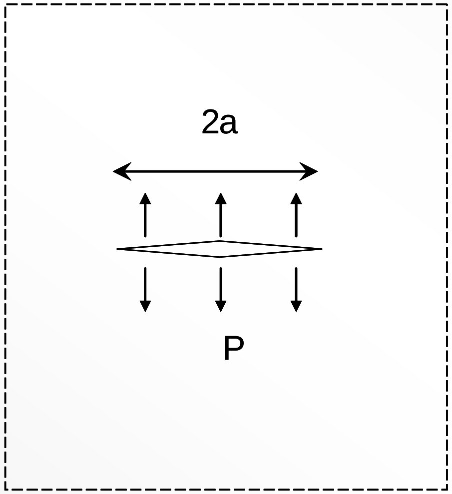
Orthotropic

$$\phi = 2\,\mathrm{Re}\left\{\psi(z_1) + \chi(z_2)\right\}$$

$z_i = x + s_i y$

Note – Material Dependencies

Specific Example a Plate



- Stress Function was solved for this particular case (Goursat Functions)
- Gave the Stress in x, y, and xy as:

$$\sigma_i = \frac{k_1}{\sqrt{2r}} f_i(s_1, s_2, \theta)$$

$$k_1 = p\sqrt{a}$$

Isotropic

$$\sigma_i = \frac{K_I}{\sqrt{2\pi r}} f_i(\theta)$$

$$K_I = p\sqrt{\pi a}$$

Strain Energy Release Rate

General Relationship:

$$G_1 = \frac{\pi k_1^2}{2} a_{22} \operatorname{Re}\left[i \left(\frac{s_1 + s_2}{s_1 s_2} \right) \right]$$

For an Orthotropic Plate:

$$G_1 = \pi k_1^2 \sqrt{\frac{a_{11} \cdot a_{22}}{2}} \cdot \sqrt{\sqrt{\frac{a_{22}}{a_{11}}} + \frac{2 \cdot a_{12} + a_{66}}{2 \cdot a_{11}}}$$

- This is the ‘exact’ solution
- Complex.
- Since K’s are nearly identical, the difference between an orthotropic and isotropic case are in G

Simplification of G

Start with…

$$G_1 = \pi k_1^2 \sqrt{\frac{a_{11} \cdot a_{22}}{2}} \cdot \sqrt{\sqrt{\frac{a_{22}}{a_{11}}} + \frac{2 \cdot a_{12} + a_{66}}{2 \cdot a_{11}}}$$

Apply…

$$E_2 = c_a \cdot E_a + c_b \cdot E_b$$

$$\frac{1}{E_1} = \frac{c_a}{E_a} + \frac{c_b}{E_b} - \frac{c_a \cdot c_b \left(\nu_a \cdot E_b - \nu_b \cdot E_a\right)^2}{E_a \cdot E_b \cdot \left(c_a \cdot E_a + c_b \cdot E_b\right)}$$

$$a_{11} := \frac{1}{E_1} \qquad a_{22} := \frac{1}{E_2} \qquad a_{12} := \frac{-\nu_{21}}{E_2} \qquad a_{66} := \frac{1}{G_{12}}$$

$$\nu_{21} = c_a \cdot \nu_a + c_b \cdot \nu_b$$

$$\frac{1}{G_{12}} = \frac{c_a}{G_a} + \frac{c_b}{G_b}$$

Simplification of G

Assume…

- Equal Volume Fractions
- E_1 can be approximated with just two terms
- Ea/Eb >> Eb/Ea

$$\sqrt{\frac{a_{11} \cdot a_{22}}{2}} = \sqrt{\frac{1}{2 \cdot E_a \cdot E_b}}$$

$$\frac{2 \cdot a_{12}}{2 \cdot a_{11}} = \frac{2(\nu_a + \nu_b)}{2 + \frac{E_a}{E_b}}$$

$$\sqrt{\frac{a_{22}}{a_{11}}} = \sqrt{\frac{4}{2 + \frac{E_a}{E_b}}}$$

$$\frac{a_{66}}{2 \cdot a_{11}} = 1 + \nu_b$$

Let $\eta = \frac{E_a}{E_b}$

Simplification of G

We get a first approximation:

$$G\_approx = \pi \cdot \frac{k_1^2}{E_a} \cdot \sqrt{\frac{\eta}{2}} \cdot \sqrt{\sqrt{\frac{4}{2+\eta}} + \frac{-2(\nu_a + \nu_b)}{2+\eta} + 1 + \nu_b}$$

This is more of a simplification. Good, but still complex.

Applying some further approximations, we get:

$$G\_approx2 = \pi \cdot \frac{k_1^2}{E_a} \cdot \eta^{0.5}$$

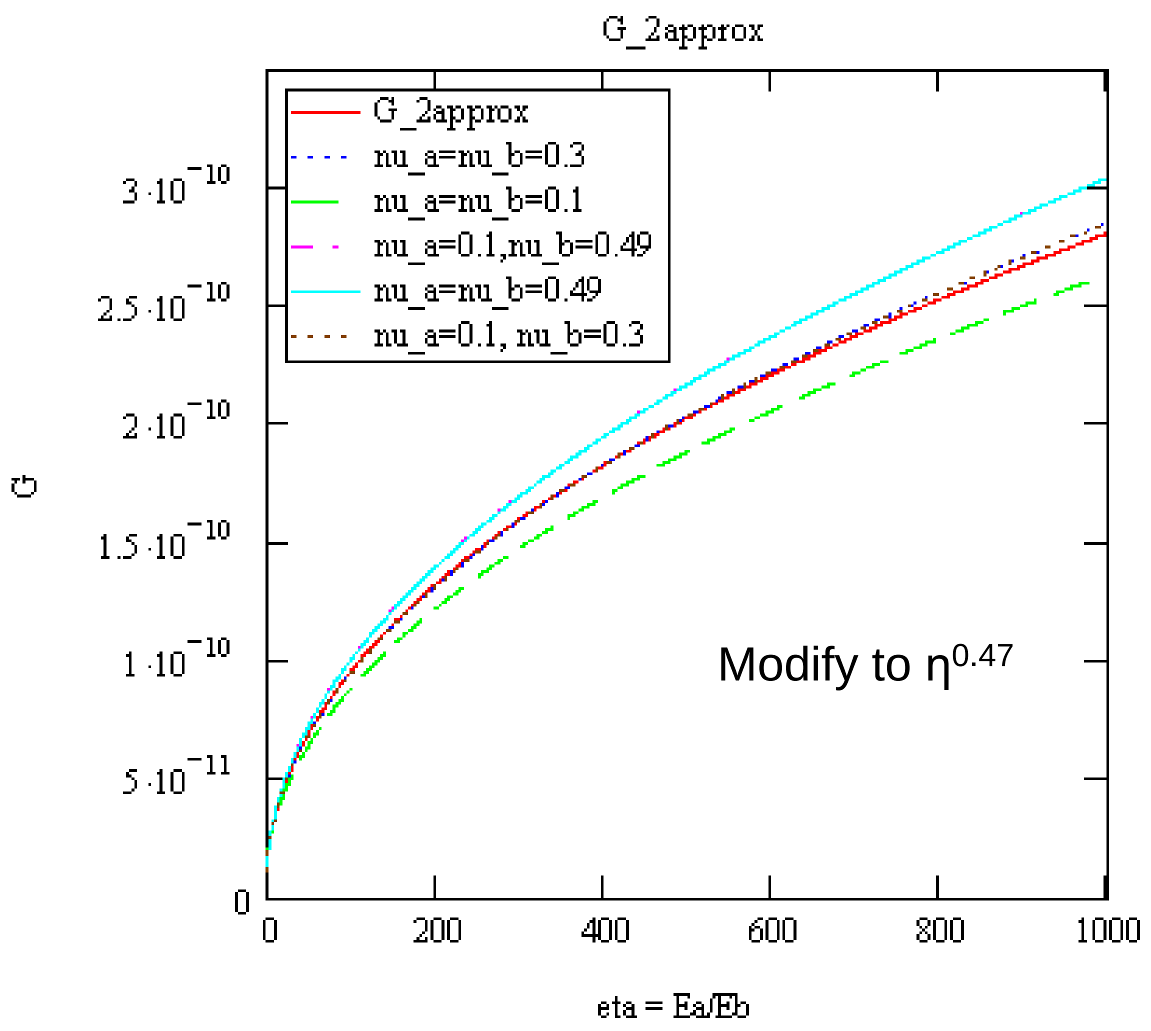
$$G = \frac{K_I^2}{E}$$

Approximation of G

- G is therefore simplified to ONE material property (Ea) and the Ea/Eb ratio.

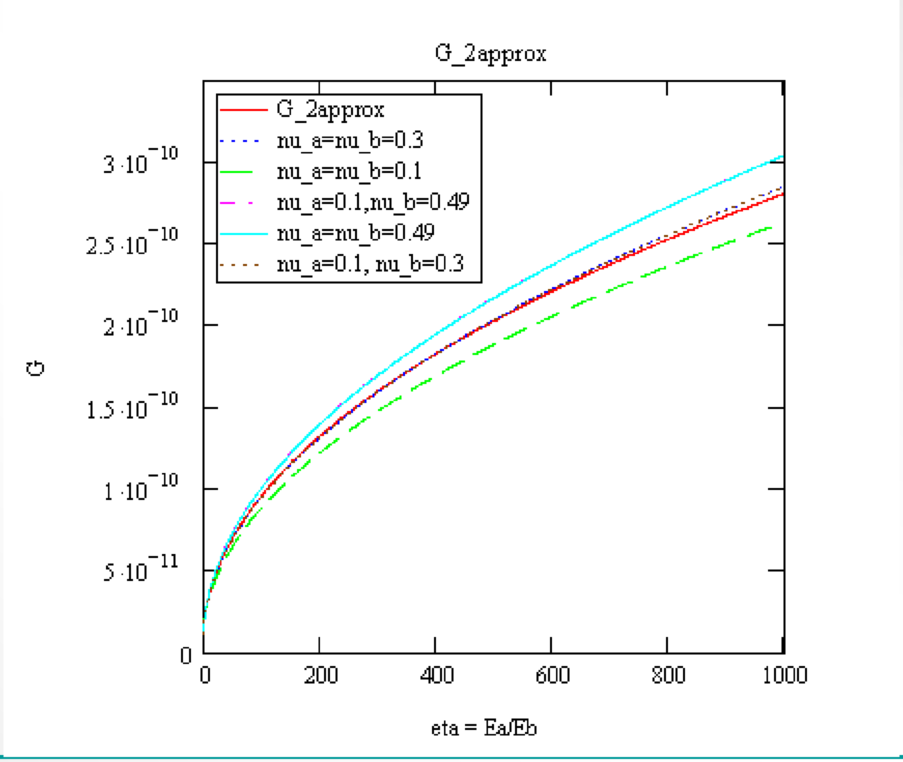
- Poisson's ratio is removed from G

Simplification of G



Conclusions about G

- The ratio of Ea/Eb has a much greater effect on G than the poisson's ratio.
- G is increased as Eb, the second phase is softened.



Finite Element Analysis

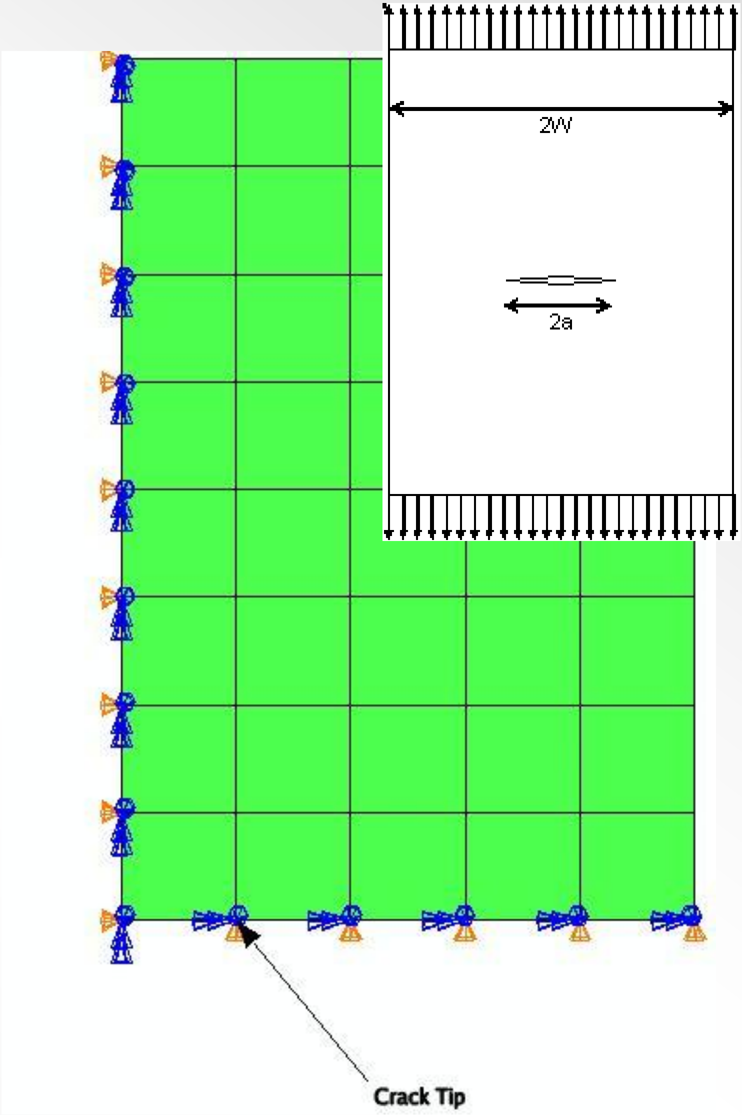
FEA Analysis - Goals

- The **effect** of the tip location in an orthotropic crack
- The **effect** of η (both layered and orthotropic)
- The **difference** between layered and orthotropic materials

A PARAMETRIC STUDY

FEA - Model

- Rectangular Model
- Upper Quadrant
- Symmetry Boundary Conditions
- Output: G
  - Used modified crack closure method

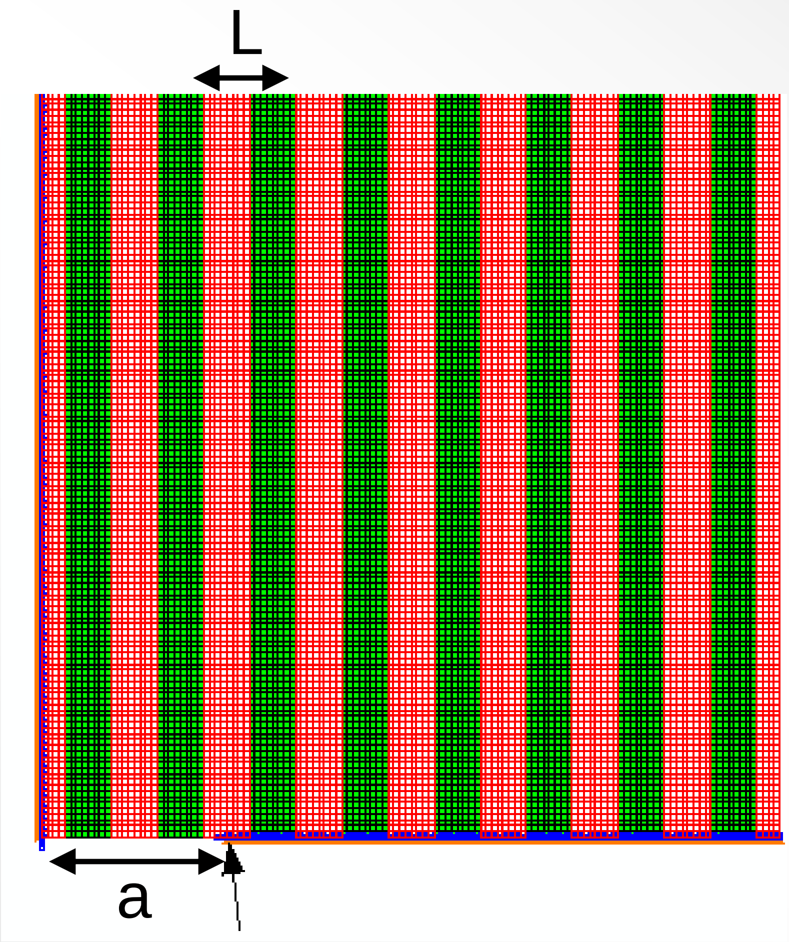


The Meshing Procedure – Generated Mesh

- Disadvantages:
  - Had to keep model simple, couldn’t have smaller mesh near the crack tip
  - Significant effort to develop an input script and post processor

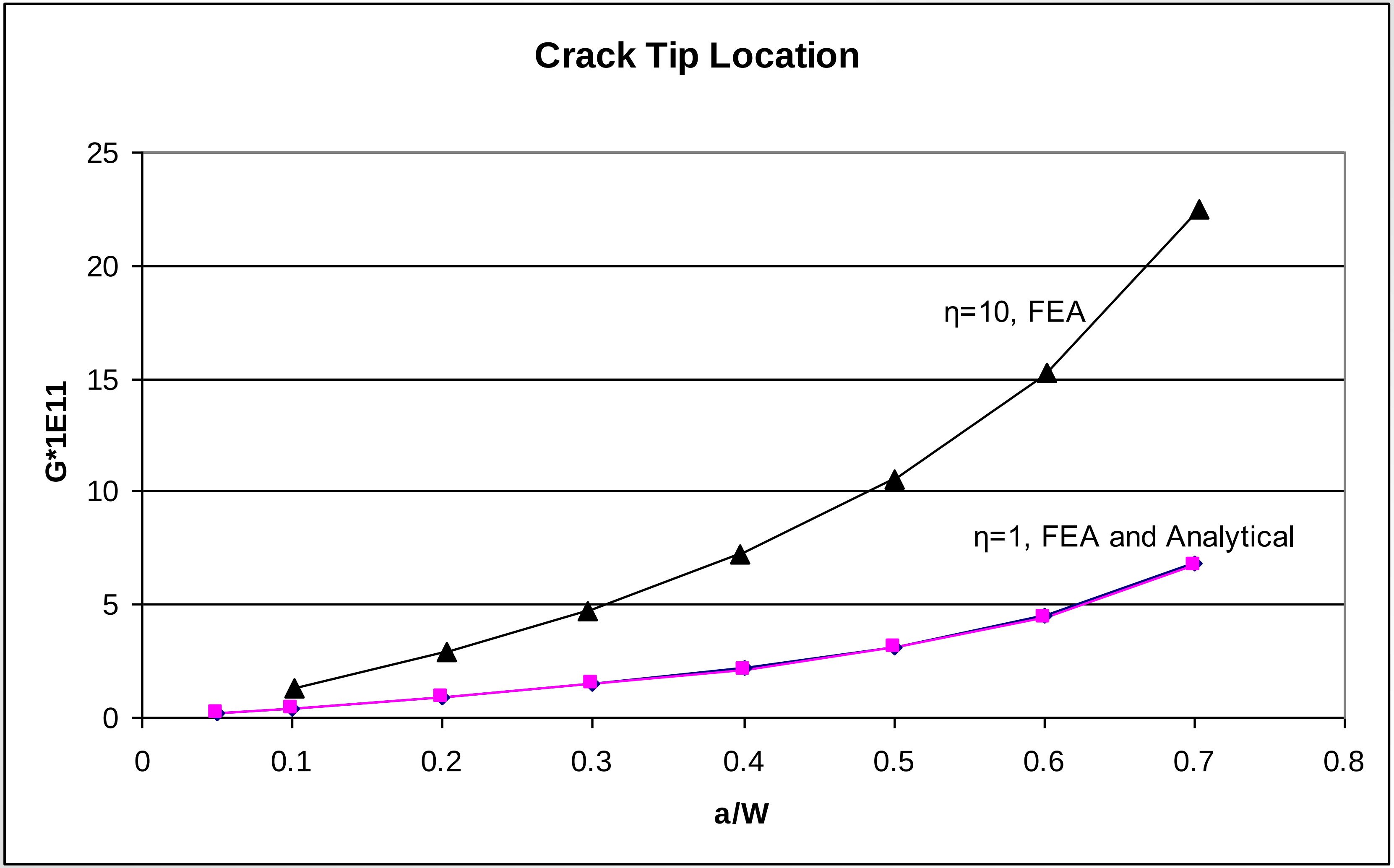
- Advantages
  - Parametric Study – more than 75 different cases run
  - Layers!

Layer Generation



While changing a/L, crack tip must be kept constant. Crack tip must also remain in the middle of a material

Finite Element Results: Crack Tip Location



Finite Element Results: Crack Tip Location

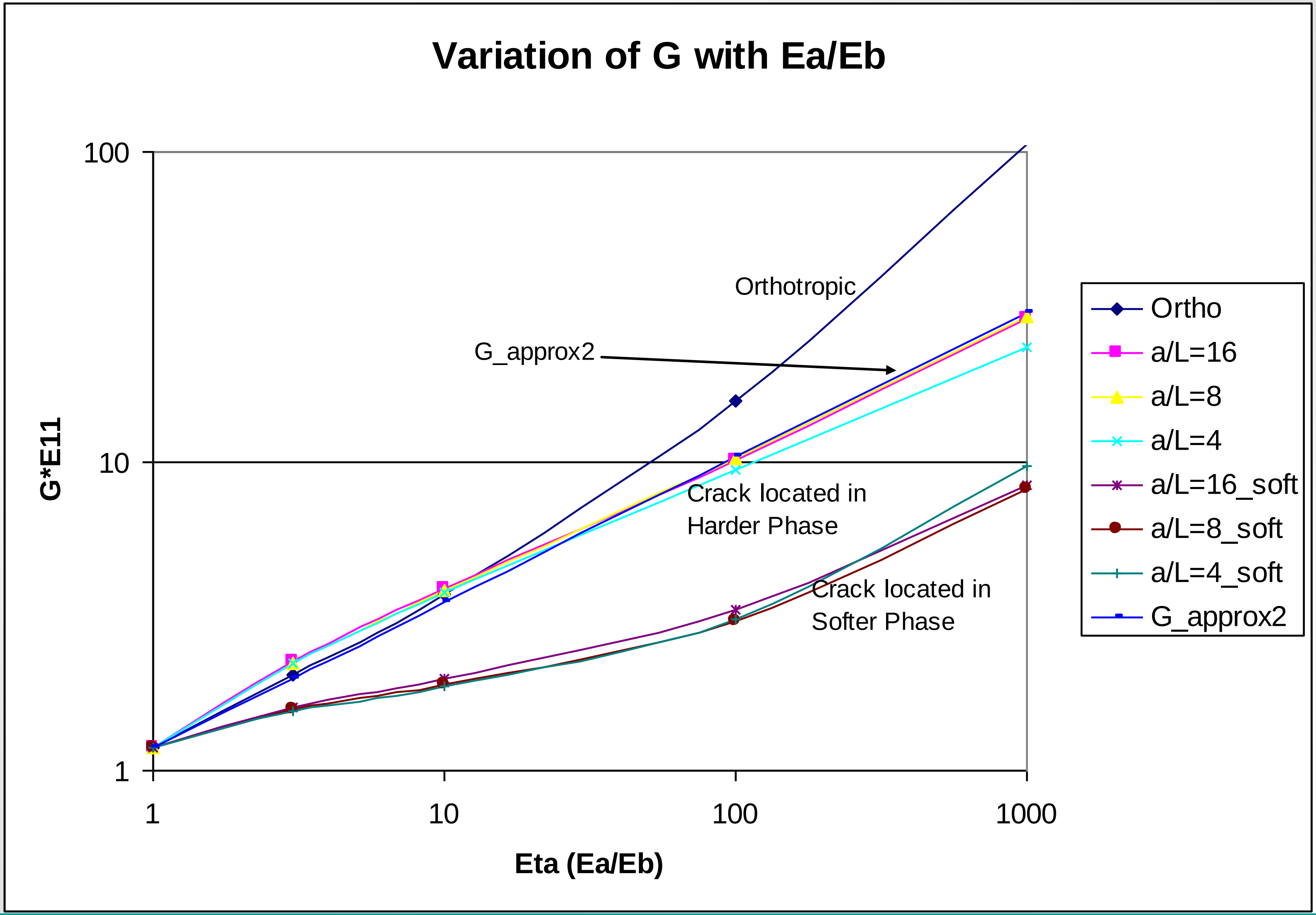
For an isotropic case, there is a geometric correction factor

$$F(a) = \sqrt{\sec\left(\frac{\pi a}{2}\right)}$$

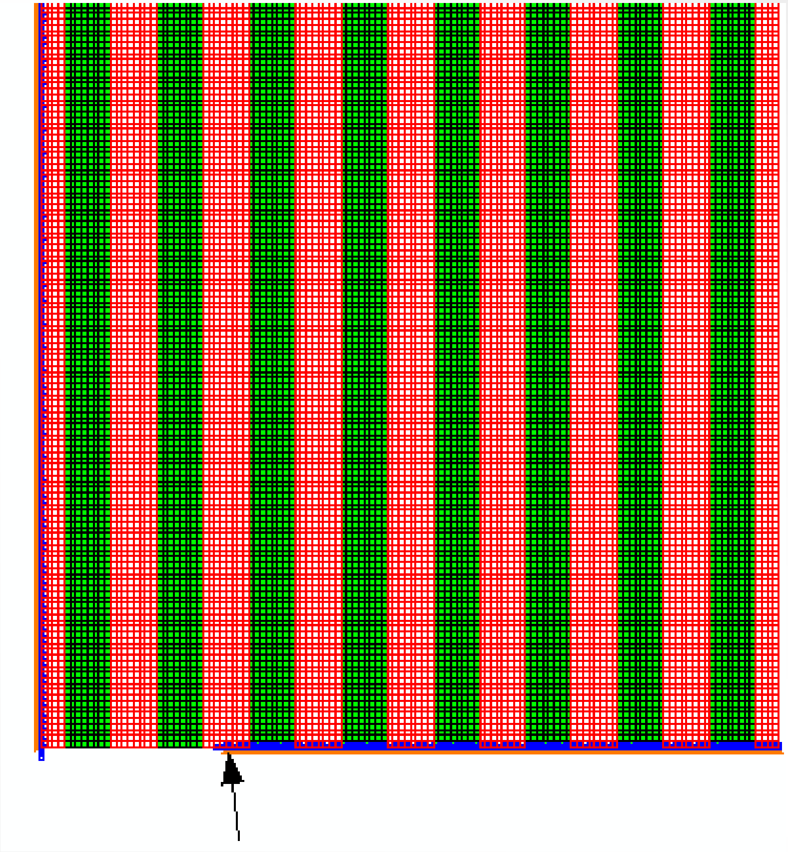
For the orthotropic case, there is a dependency also on η

$$F(a, \eta) = ?$$

Effect of η on G



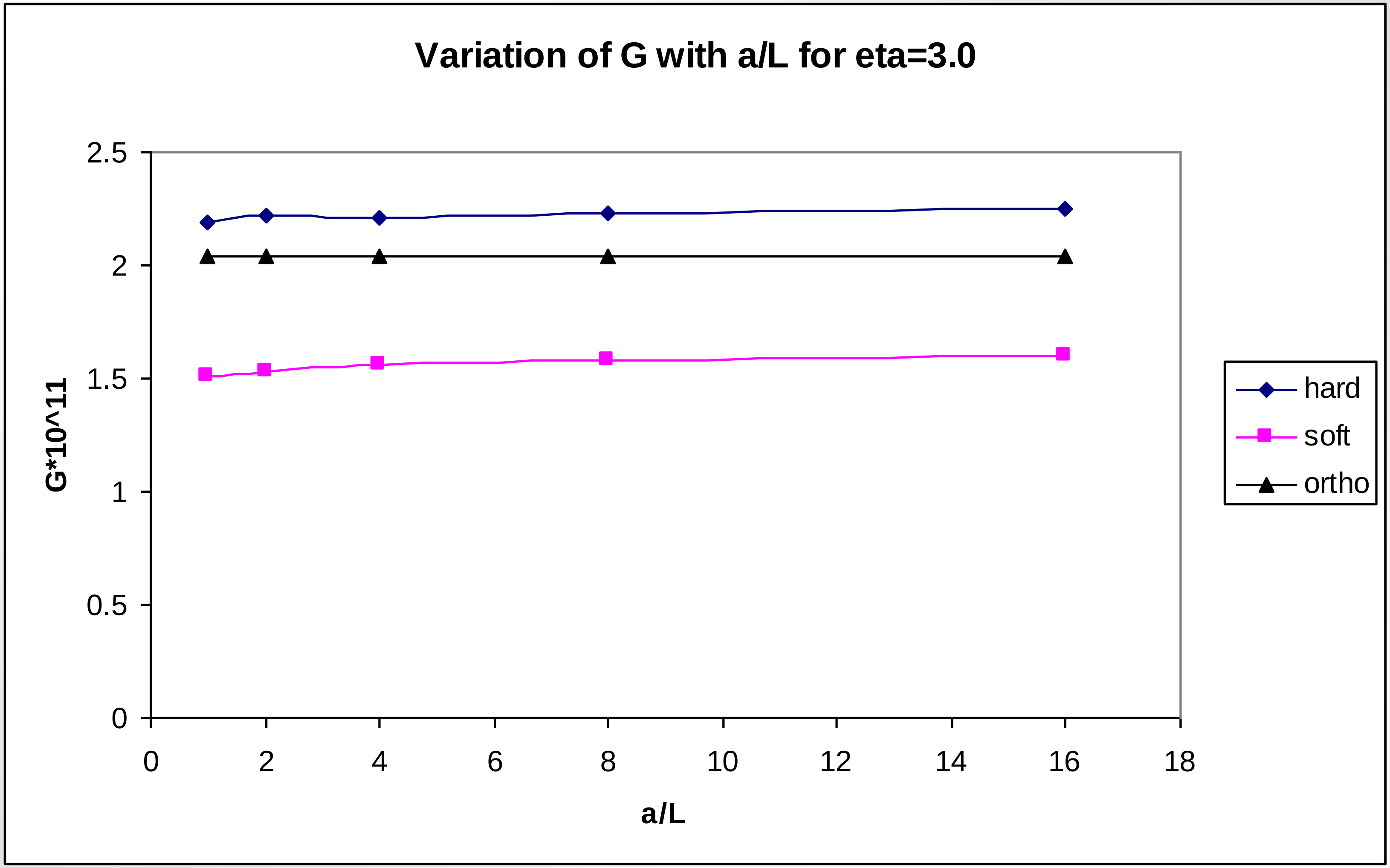
Hard vs. Soft



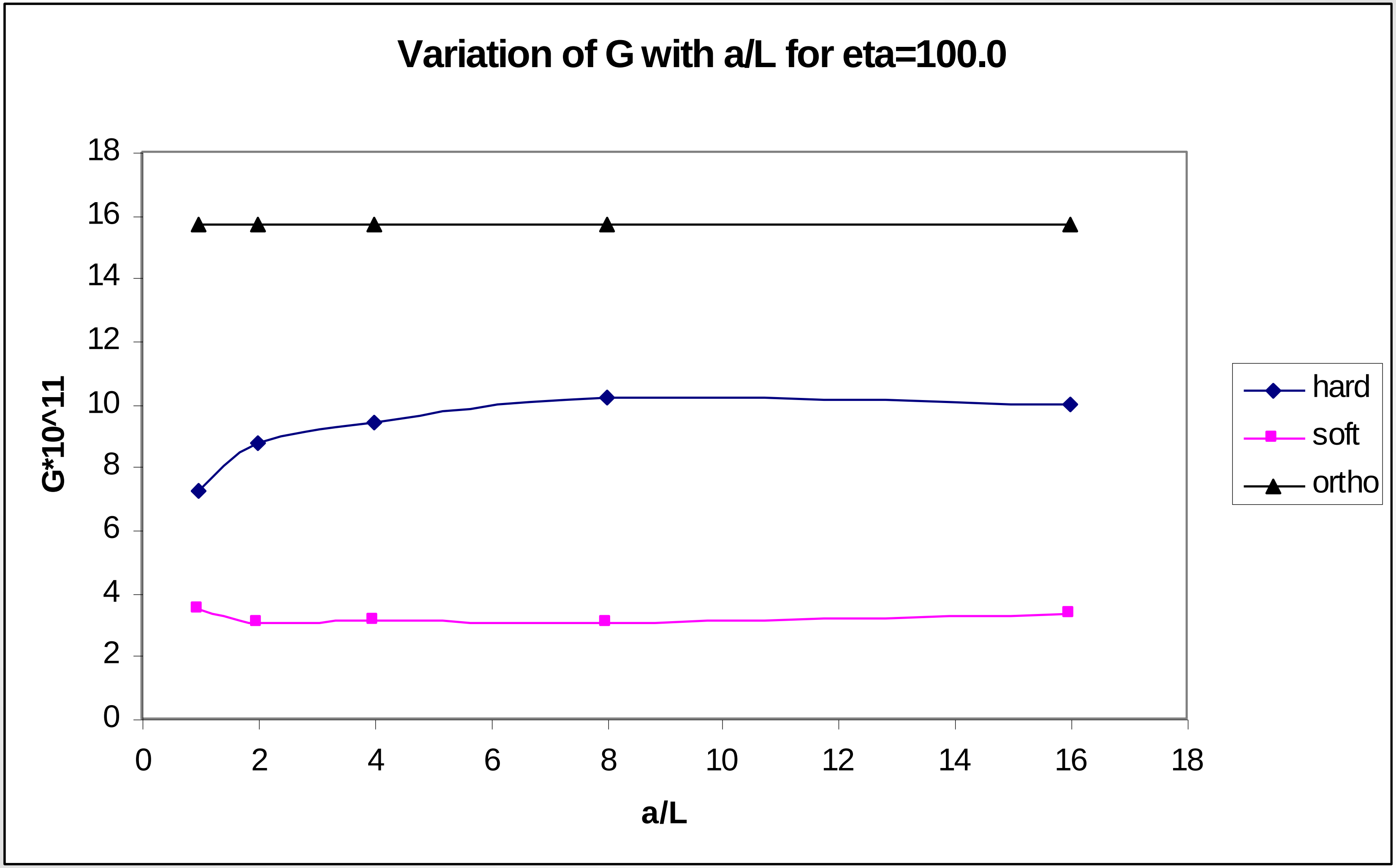
Effect of η on G

- η shows a strong effect on G for both layered and orthotropic materials
  - Validates the analytical work
- a/L has a much weaker effect

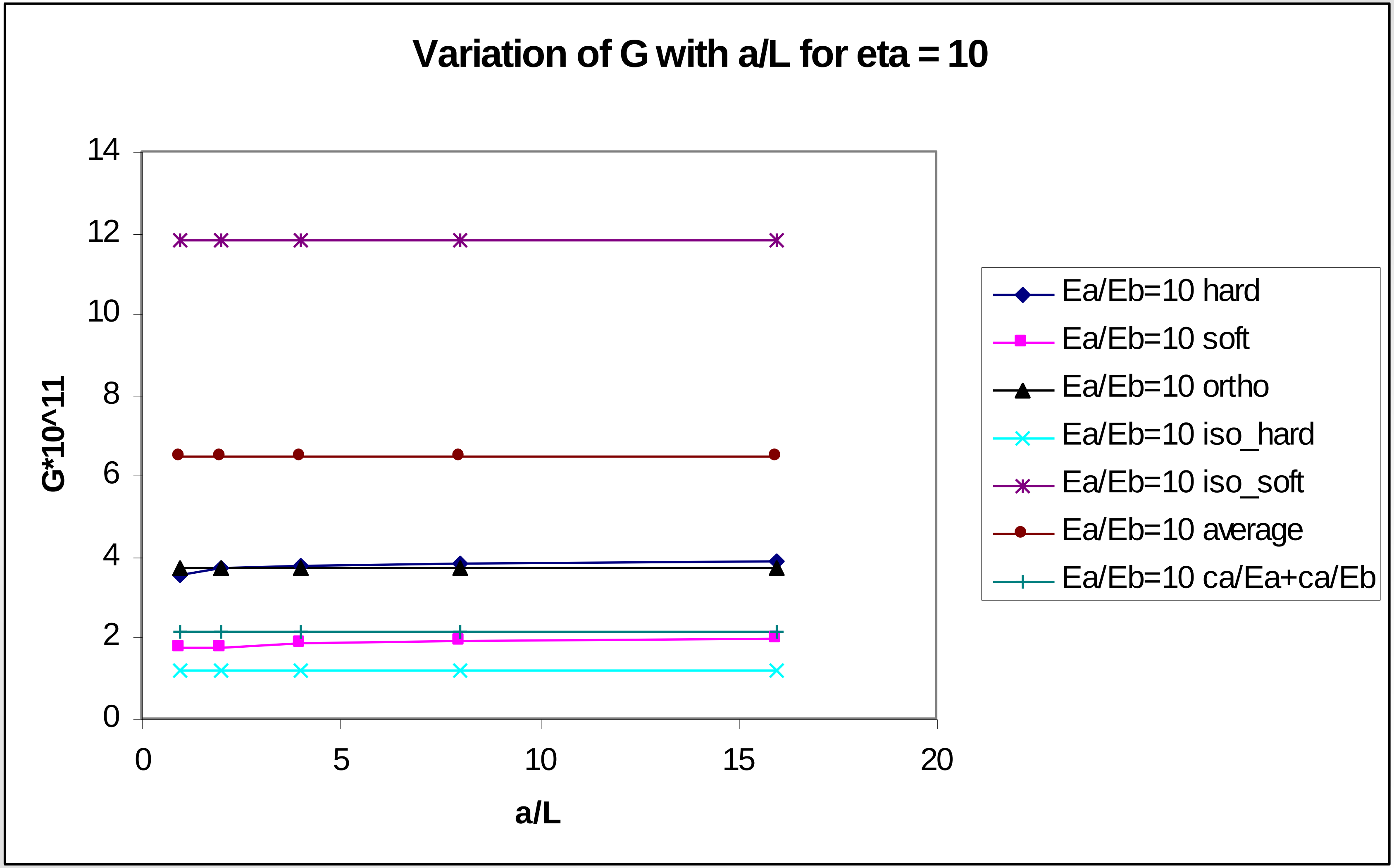
Layered vs. Orthotropic



Layered vs. Orthotropic



Layered vs. Orthotropic - GUESSES



Layered vs. Orthotropic: Conclusions

- G is not strongly dependent on a/L, at least within the bounds of 1 to 16
- Whether the crack tip is in the soft or the harder material strongly affects the resulting G.
- There does not appear to be a physically based method for predicting G in the layered material over a realistic range of η.

Discussion/Future Work

Discussion

- η plays a crucial role in the effect of orthotropic and layered materials. This can be approximated as $\eta^{0.47}$.
- Layered and Orthotropic Materials cannot be compared, yet.

Questions?

Acknowledgements

- Labmates for help with Abaqus!
- Audience!
- Professor Sun

References

1. Sih, G.C. and H. Liebowitz, *Mathematical Theories of Brittle Fracture*, in *Fracture, An Advanced Treatise*, H. Liebowitz, Editor. 1968, AP: New York.
2. Sun, C.T., *Fracture Mechanics*. 2007, Purdue University.
3. Sun, C.T., *Composites Class Notes*. 2007, Purdue University.

References – Pictures

NDT Resource Center: http://www.ndt-ed.org/EducationResources/CommunityCollege/Materials/Structure/composite.htm